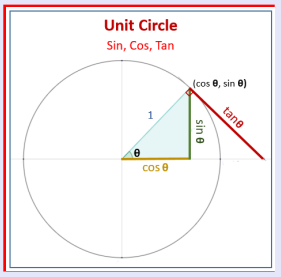
Trigonometry

Lecture 27



Feb 19-8:47 AM

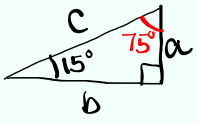
Show

$\sin 45^\circ + \sin 15^\circ = \sin 75^\circ$

Hint: $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

$\sin 45^\circ + \sin 15^\circ = 2 \sin \frac{45^\circ+15^\circ}{2} \cos \frac{45^\circ-15^\circ}{2}$

$= 2 \sin 30^\circ \cos 15^\circ$



$= 2 \cdot \frac{1}{2} \cos 15^\circ = \cos 15^\circ$

$= \frac{b}{c} = \sin 75^\circ$

Co functions of Complementary angles are equal.

$\sin 10^\circ = \cos 80^\circ$

$\tan 25^\circ = \cot 65^\circ$

$\sec 40^\circ = \csc 50^\circ$

$\sin 42^\circ = \cos 48^\circ$

Oct 15-10:27 AM

Verify $\dfrac{\sin 4x}{\sin x} = 4\cos x \cos 2x$

Hint: $4x = 2(2x)$

$$\frac{\sin 4x}{\sin x} = \frac{\sin[2(2x)]}{\sin x} = \frac{2\sin 2x \cos 2x}{\sin x}$$

$$= \frac{2 \cdot 2\sin x \cos x \cdot \cos 2x}{\sin x}$$

$$= 4\cos x \cos 2x$$

Oct 15-10:35 AM

Verify $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$3x = 2x + x$

$\tan 3x = \tan(2x + x) = \dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \dfrac{\frac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \frac{2\tan x}{1-\tan^2 x} \cdot \tan x}$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

LCD $= 1 - \tan^2 x$

$= \dfrac{2\tan x + \tan x(1 - \tan^2 x)}{1 - \tan^2 x - 2\tan^2 x} = \dfrac{2\tan x + \tan x - \tan^3 x}{1 - 3\tan^2 x}$

$= \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

Oct 15-10:39 AM

Verify $\tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right) = \frac{1+\sin x}{1-\sin x}$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1-\cos 2x}{2}}{\frac{1+\cos 2x}{2}}$

$\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$

So

$\tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right) = \frac{1-\cos 2\left(\frac{x}{2}+\frac{\pi}{4}\right)}{1+\cos 2\left(\frac{x}{2}+\frac{\pi}{4}\right)}$

$= \frac{1-\cos\left(x+\frac{\pi}{2}\right)}{1+\cos\left(x+\frac{\pi}{2}\right)} = \frac{1-\left[\cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}\right]}{1+\left[\cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}\right]} = \frac{1+\sin x}{1-\sin x}$

($\cos\frac{\pi}{2} \to 0$, $\sin\frac{\pi}{2} \to 1$)

Oct 15-10:47 AM

Verify

$$\frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \tan 3x$$

$\sin x + \sin 5x = 2\sin\frac{x+5x}{2}\cos\frac{x-5x}{2} = 2\sin 3x \cos(-2x)$

$= 2\sin 3x \cos 2x$

$\cos x + \cos 5x = 2\cos\frac{x+5x}{2}\cos\frac{x-5x}{2} = 2\cos 3x \cos 2x$

$$\frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \frac{2\sin 3x\cos 2x + \sin 3x}{2\cos 3x \cos 2x + \cos 3x}$$

$$= \frac{\sin 3x\left[2\cos 2x + 1\right]}{\cos 3x\left[2\cos 2x + 1\right]}$$

$= \tan 3x$

Oct 15-10:58 AM

Verify

$$\tan y = \frac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)}$$

$\sin x\cos y - \cos x\sin y$

$$= \frac{\sin x\cos y + \cos x\sin y - \sin x\cos y + \cos x\sin y}{\cos x\cos y - \sin x\sin y + \cos x\cos y + \sin x\sin y}$$

$$= \frac{2\cos x\sin y}{2\cos x\cos y} = \frac{\sin y}{\cos y} = \tan y$$

Oct 15-11:06 AM

Show $\dfrac{\sin x + \sin y}{\cos x + \cos y} = \tan\left(\dfrac{x+y}{2}\right)$

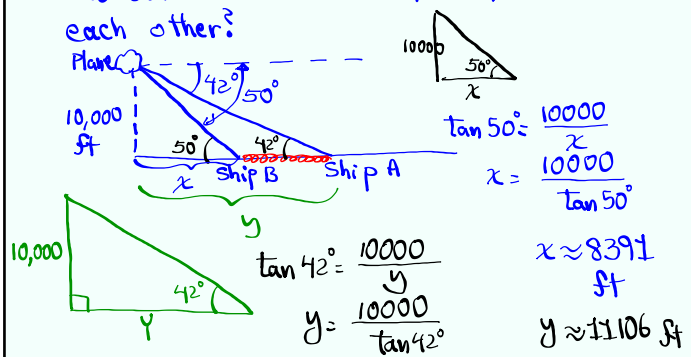
$$\frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}} = \frac{\sin\frac{x+y}{2}}{\cos\frac{x+y}{2}}$$

Oct 15-11:11 AM

A plane is flying horizontally at 10,000 ft high.

It spots two ships with angle of depression of 42° and 50°.

How far are these ships apart from each other?



$\tan 50° = \frac{10000}{x}$

$x = \frac{10000}{\tan 50°}$

$x \approx 8391$ ft

$\tan 42° = \frac{10000}{y}$

$y = \frac{10000}{\tan 42°}$

$y \approx 11106$ ft

Distance = $11106 - 8391 =$ ____ ft

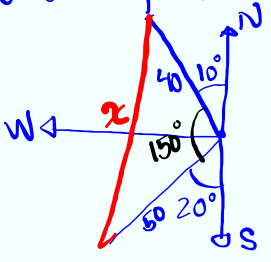
Oct 15-11:14 AM

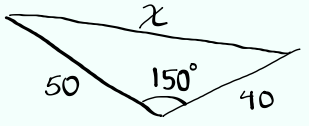
Two ships left same port at same time.

Ship A : 40 miles @ N 10° W

Ship B : 50 miles @ S 20° W

How far apart are they now?





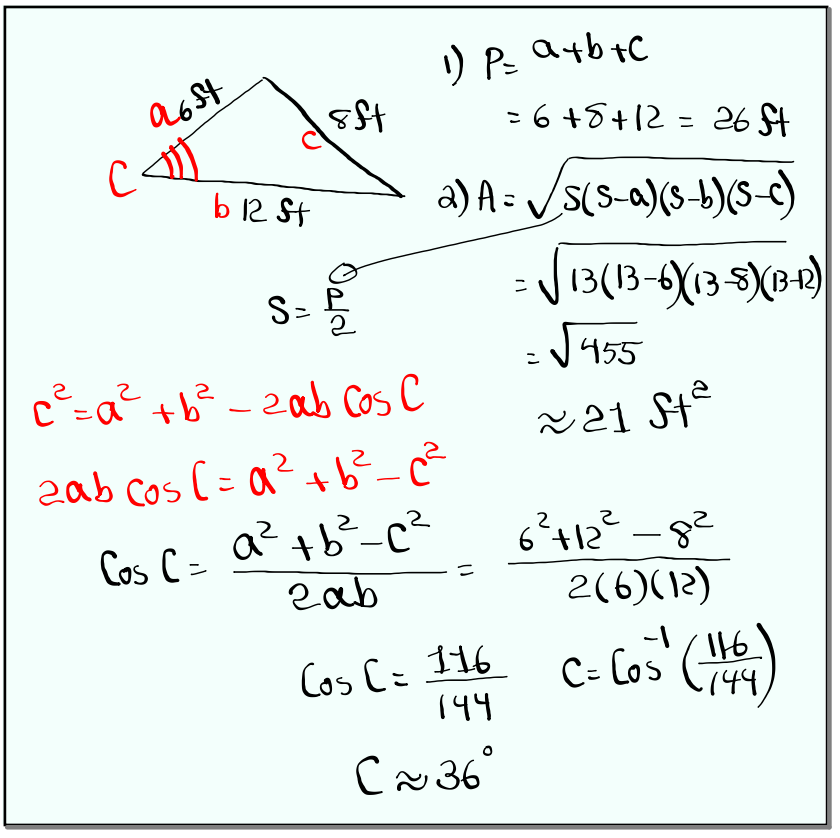
Law of Cosines

$x^2 = 40^2 + 50^2 - 2 \cdot 40 \cdot 50 \cos 150°$

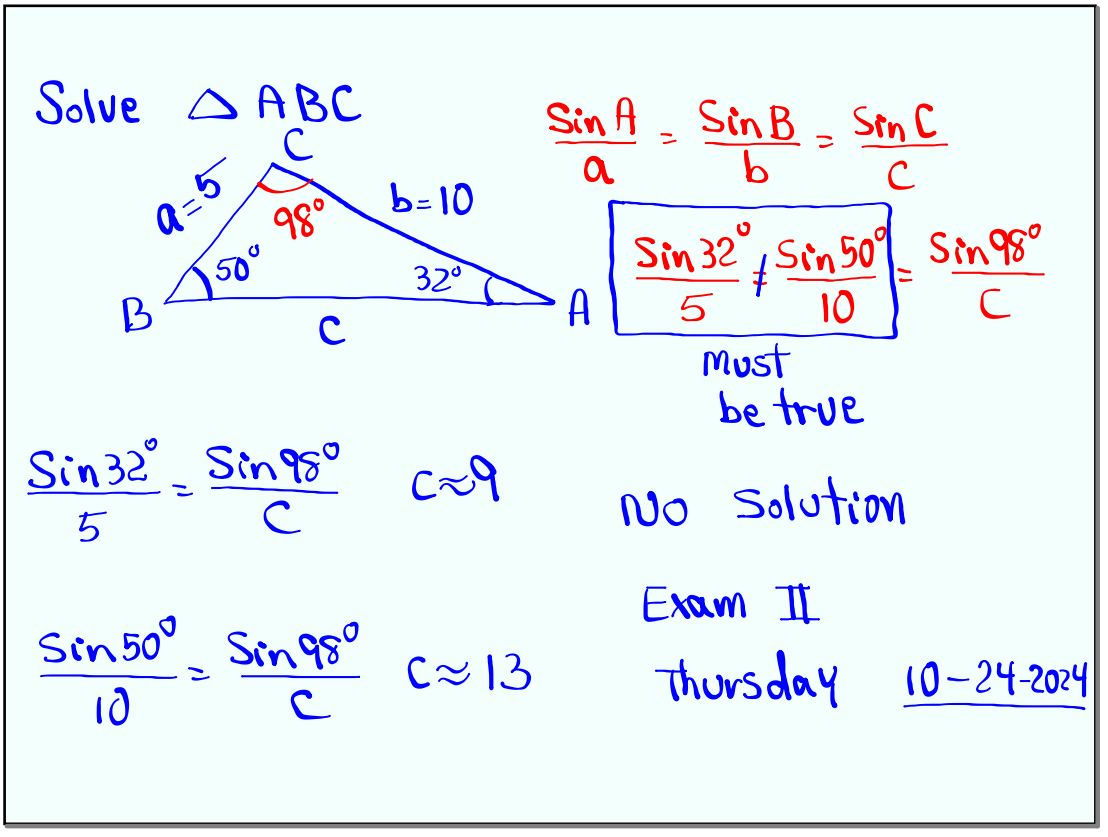
$x^2 \approx 7564$

$x \approx 87$ miles

Oct 15-11:22 AM



Oct 15-11:29 AM



Oct 15-11:35 AM